Coherently Driven Micromaser Pumped by Atoms in Superposition States

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By a Monte Carlo wave function approach we describe the dynamics of a coherently driven micromaser pumped by coherently prepared atoms. The system can exhibit nonclassical effects such as quadrature squeezing, sub-Poissonian photonstatistics, and Rabi oscillations revival. The interplay between driving field and polarised atoms allows manipulating both the photon number and the phase of the cavity field. By varying either the atomic transit time or the injected field intensity, the coherence of the system dynamics can be enhanced or even suppressed, whereas the induced cavity field phase can be shifted.

Key words: Micromaser; Nonclassical Field States; Quantum Trajectories.

1. Introduction

The coherent Jaynes-Cummings (JC) interaction between a single two-level atom and a single cavity mode is at the heart of the one-atom maser and laser dynamics [1] and of the occurrence of nonclassical effects in cavity quantum electrodynamics, up to the recent observations of trapping states and Fock states of the cavity field [2]. We investigated the dynamics of a micromaser driven by a resonant coherent field [3] by a Monte Carlo wave function (MCWF) approach [4]. In this paper we consider the coherently driven micromaser when the atoms are prepared in *superposition states* rather than incoherently excited. In this case the atoms are polarised and can transfer a phase to the cavity mode, just like the injected signal. The combination of polarised atoms and driving field can thus induce interesting dynamical effects [5].

We present a model of the system based on a Master Equation for the cavity mode density operator, where the jump operators have a transparent physical interpretation. By the MCWF method we investigate system dynamics, using either the atomic transit time or the driving field intensity as control parameters. Besides the occurrence of nonclassical behaviours like in the case with fully excited atoms, we find peculiar effects due to the injection of coherently prepared atoms. In particular, the coherence of system dynamics can be enhanced or dramatically reduced, depending on atomic state preparation, and intriguing phase competition effects can show up.

2. The Model

Let us consider a micromaser driven by a resonant coherent field with amplitude ε (taken real without loss of generality), and pumped by atoms prepared in a superposition state

$$|\Psi_{\mathrm{A}}\rangle = \alpha|e\rangle + \beta|g\rangle, \tag{1}$$

where $|e\rangle$ ($|g\rangle$) is the upper (lower) Rydberg state and $|\alpha|^2+|\beta|^2 = 1$. The system dynamics can be described by the following Master Equation (ME) for the cavity mode density operator $\rho(t)$, written in the Lindblad form and in the interaction picture:

$$\dot{\rho}(t) = \sum_{k=1}^{4} \left\{ \hat{C}_k \rho(t) \hat{C}_k^+ - \frac{1}{2}\left[\hat{C}_k^+ \hat{C}_k \rho(t) + \rho(t) \hat{C}_k \hat{C}_k^+\right] \right\}, \tag{2}$$

where the jump operators are

$$\hat{C}_1 = \sqrt{N_{\mathrm{ex}}}\hat{D}(\varepsilon/g)\Big[\alpha \cos\left(g t_{\mathrm{int}}\sqrt{\hat{N}+1}\,\right) - \beta \frac{\sin\left(g t_{\mathrm{int}}\sqrt{\hat{N}+1}\,\right)}{\sqrt{\hat{N}+1}}\hat{a}\Big]\hat{D}^+(\varepsilon/g),$$

$$\hat{C}_2 = \sqrt{N_{ex}}\hat{D}(\varepsilon/g)\Big[\alpha\frac{\sin\left(gt_{int}\sqrt{\hat{N}}\right)}{\sqrt{\hat{N}}}\hat{a}^+ \qquad (3)$$

$$+\beta\cos\left(gt_{int}\sqrt{\hat{N}}\right)\Big]\hat{D}^+(\varepsilon/g),$$

$$\hat{C}_3 = \sqrt{1+\bar{n}}\hat{a},\ \hat{C}_4 = \sqrt{\bar{n}}\hat{a}^+.$$

In (2), (3) t and N_{ex} are the dimensionless evolution time and atomic pump rate, g is the JC coupling constant, t_{int} the interaction (atomic transit) time, $\bar{n}$ the mean thermal photon number, $\hat{D}(\varepsilon/g)$ the unitary displacement operator. The ME (2) describes the time evolution of the cavity mode density operator as a continuous, non-unitary evolution interrupted by the occurrence of jumps, described by the operators (3), due to the interaction with an atom ($\hat{C}_1, \hat{C}_2$) or with the environment ($\hat{C}_3, \hat{C}_4$). The operator $\hat{C}_3$ ($\hat{C}_4$) describes the absorption (emission) of a photon induced by the environment. The operator $\hat{C}_1$ ($\hat{C}_2$) describes the driven JC evolution in which one atom leaves the cavity in the upper (lower) state after absorbing (emitting) one photon, with amplitude β (amplitude α) *and* in the same state as at the input, with amplitude α (amplitude β). By the MCWF technique we can calculate the time evolution and the steady-state values of all relevant cavity mode and atomic observables and distributions, including all main physical effects besides temperature, such as atomic velocity spread $\Delta v/v$, collective effects, level decays and detector efficiency [3, 6]. A first relevant example is the probability that the atom leaves the cavity in the upper state, because in micromaser systems the populations of the Rydberg levels are measurable quantities by the field ionisation technique. If we neglect thermal effects, its expression is

$$P_e^{out}(t) = \sum_{n=0}^{\infty}\Big\{|\alpha|^2\cos^2\big(\frac{\Omega_n}{2}t_{int}\big)\rho_{nn}(t)$$

$$+|\beta|^2\sin^2\big(\frac{\Omega_n}{2}t_{int}\big)\rho_{n+1,n+1}(t) \qquad (4)$$

$$-\sin(\Omega_n t_{int})\mathrm{Re}\big[\alpha\beta^*\rho_{n,n+1}(t)\big]\Big\},$$

where $\Omega_n = 2g\sqrt{n+1}$ is the n-photon Rabi frequency and $\rho_{m,n} = 000$. $\rho_{mn} = \langle m|\rho|n\rangle$ is a cavity mode density matrix element in the Fock states basis. The interference term depends on the real part of the product of the cavity mode coherences with the expectation value of the atomic lowering operator, $< |g\rangle\langle e| >= \alpha\beta^*$, that is proportional to the dipole moment of the atoms injected in the superposition state (1). Hence it depends on the relative phase between atoms and cavity field. We shall consider also the steady-state phase distribution of the cavity field [7]

$$P(\vartheta) = \frac{1}{2\pi}\sum_{m,n=1}^{\infty}\rho_{mn}\exp[i(m-n)\vartheta]. \qquad (5)$$

3. Results and Discussion

We integrate the ME (2) by the MCWF approach, with parameter values taken from the recent micromaser trapping state experiment [2]: $N_{ex} = 7$, $g =$ 39 kHz, $\bar{n} = 0.054$, $\Delta v/v = 0.03$. The cavity mode is initially in a thermal state not far from the vacuum state; each atom is injected in the superposition state (1), that now we rewrite as

$$|\Psi_A\rangle = |\alpha||e\rangle + |\beta|\exp(i\varphi_A)|g\rangle. \qquad (6)$$

The real part of the expectation value of atomic dipole moment in the state (6) is maximum for $|\alpha| = |\beta| = 1/\sqrt{2}$, $\varphi_A = 0$, and vanishes for $\varphi_A = \pi/2$. We take as control parameters for the system dynamics the dimensionless interaction time θ_{int} and the equivalent number of injected photons n_ε:

$$\theta_{int} = gt_{int}\sqrt{N_{ex}},\ n_\varepsilon = 4\varepsilon^2/\gamma_c^2, \qquad (7)$$

where γ_c is the photon decay rate.

In Fig. 1 we show the steady-state atomic population inversion

$$I = P_g^{out} - P_e^{out}, \qquad (8)$$

where P_g^{out} (P_e^{out}) is the occupation probability of the lower (upper) state out of the cavity. In the coherently driven micromaser with fully excited atoms we see three damped Rabi oscillations, followed by collapse and revival. For atoms injected in a pure state ($|\alpha| = |\beta| = 1/\sqrt{2}$) we find an extended coherence (with four oscillations) for $\varphi_A = 0$, the maximally polarised state; on the contrary, coherence is strongly reduced for $\varphi_A = \pi/2$, where we see just a single oscillation. For different values of the atomic parameters

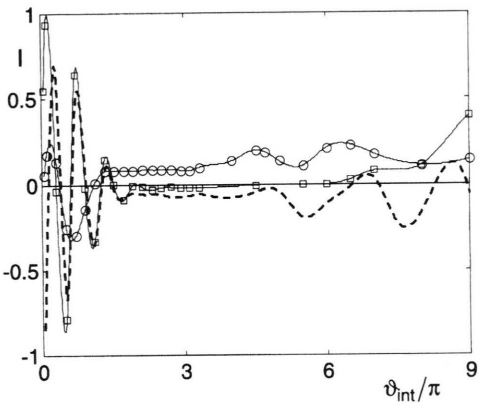

Fig. 1. Steady-state atomic population inversion I (8) vs. dimensionless interaction time $\theta_{\rm int}$, at fixed driving field intensity (n_ε = 20). Atoms injected in superposition state (6) with $|\alpha| = |\beta| = 1/\sqrt{2}$ and $\varphi_{\rm A} = 0$ (line with symbols □), or $\varphi_{\rm A} = \pi/2$ (line with symbols ∘); fully excited atoms (dashed line).

$|\alpha|$, $|\beta|$, $\varphi_{\rm A}$ ($\varphi_{\rm A} \neq \pi/2$), the behaviour of population inversion is qualitatively similar to the above considered case of maximum atomic polarisation. A nice revival effect is observed, e. g., in the case $|\alpha| = \sqrt{3}/2$, $|\beta| = 1/2$, $\varphi_{\rm A} = 0$, starting at $\theta_{\rm int}/\pi \approx 7$. Coherence enhancement and suppression are manifest in the behaviour of the steady-state cavity mode mean photon number, not reported here. The suppression of coherence for $\varphi_{\rm A} = \pi/2$ can be explained as an effective decoupling of the external field from the system [5].

These effects of coherence enhancement or suppression can affect the observation of peculiar quantum effects such as quadrature squeezing and sub-Poissonian photonstatistics. For instance, if we consider the steady-state variance $(\Delta\hat{x}_1)^2$ of the cavity mode quadrature amplitude $\hat{x}_1 = (\hat{a} + \hat{a}^+)/2$, we find that the range of interaction times for the occurrence of squeezing (up to $\theta_{\rm int}/\pi \approx 2.4$) with atoms prepared in the maximum polarisation state is *more than doubled* relative to the case with fully excited atoms. In the same range the steady-state cavity mode photonstatistics oscillate between super and sub-Poissonian. By contrast, with atomic phase $\varphi_{\rm A} = \pi/2$, no squeezing occurs and the photonstatistics is super-Poissonian at any interaction time. We recall that in micromaser systems there is a connection between cavity mode photonstatistics and atom statistics, that can be the object of experimental measurements [8].

Let us now focus on the results on phase competition effects due to the *simultaneous* presence of the

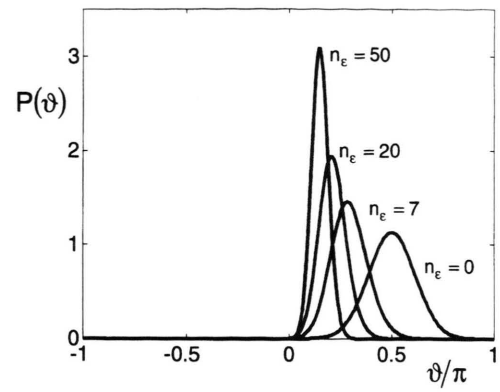

Fig. 2. Steady-state cavity mode phase distribution $P(\vartheta)$ (5) at fixed interaction time $\theta_{\rm int}$, for different driving field intensities (injected photon numbers n_ε). Atoms injected in superposition state $|\alpha| = \sqrt{3}/2$, $|\beta| = 1/2$, $\varphi_{\rm A} = \pi/2$.

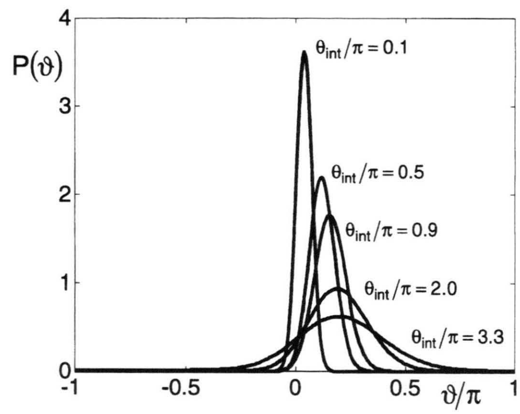

Fig. 3. Steady-state cavity mode phase distribution $P(\vartheta)$ at fixed driving field intensity (n_ε = 20), and different interaction times. Atoms injected in superposition state $|\alpha| = |\beta| = 1/\sqrt{2}$, $\varphi_{\rm A} = \pi/4$.

external field and the polarised atomic beam. In Fig. 2 we see the steady-state cavity mode phase distribution $P(\vartheta)$, defined in (5), for atoms injected in a superposition state with $|\alpha| = \sqrt{3}/2$, $|\beta|$ =1/2, $\varphi_{\rm A} = \pi/2$, at fixed interaction time ($\theta_{\rm int}/\pi = 1/2$) and for different driving field intensities. In the absence of the external field ($n_\varepsilon = 0$) the cavity mode shows a preferred phase at the value induced by the atoms ($\varphi_{\rm A} = \pi/2$). In the presence of the coherent field, whose reference phase is $\varphi_\varepsilon = 0$, the preferred phase of the cavity mode shifts from $\varphi_{\rm A}$ towards φ_ε for increasing values of n_ε. In this process $P(\vartheta)$ becomes more and more peaked. Here we see an example of manipulation of the cavity field phase: initially a phase is generated by the atoms crossing the cavity, then this phase can be shifted by turning the driving field on and changing its intensity.

In Fig. 3 we show the cavity mode phase distribution $P(\vartheta)$ in a kind of complementary situation, where the external field intensity is fixed ($n_\varepsilon = 20$) and we investigate the steady-state behaviour for different interaction times. The atoms are injected in a superposition state with $|\alpha| = |\beta| = 1/\sqrt{2}$ and $\varphi_A = \pi/4$. For short interaction times ($\theta_{int}/\pi = 0.1$) the phase distribution is sharply peaked at a value only slightly larger than the driving field phase ($\varphi_\varepsilon = 0$). For longer interaction times $P(\vartheta)$ shifts to larger phase values, i.e., towards the atomic phase ($\varphi_A = \pi/4$), and becomes more and more broadened. The atoms induce a shift of the cavity field phase originally generated by the driving field, however decoherence sets a limit to phase variation for increasing interaction times. Cavity field phase measurements can be related to measurements on the outgoing atoms by a Ramsey configuration with the micromaser cavity inserted between a first microwave cavity for coherent atomic excitation, and a second one prior to measurements [9].

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